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# Dispersion Relations and the Nucleon Polarizability\*

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## Abstract

Recent experimental results on the proton and neutron polarizabilities are examined from the point of view of backward dispersion relations. Results are found to be in reasonable agreement with the measured values. A rigorous relationship between the nucleon and pion polarizabilities is derived and shown to be in excellent agreement with several models.

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# 1 Introduction

There has been a good deal of recent experimental activity involved with measurement of the electric and magnetic polarizabilities of the nucleon, labeled  $\bar{\alpha}_E$  and  $\bar{\beta}_M$ , respectively. As a result there now exist reasonably precise values for both the proton [1, 2, 3] and the neutron [4]

$$\begin{aligned}\bar{\alpha}_E^p &= (11.3 \pm 0.7 \pm 0.8) & \bar{\beta}_M^p &= (2.9 \mp 0.7 \mp 0.8) \\ \bar{\alpha}_E^n &= (12.6 \pm 1.5 \pm 2.0) & \bar{\beta}_M^n &= (3.2 \mp 1.5 \mp 2.0).\end{aligned}\tag{1}$$

Above and hereafter all polarizabilities are quoted in units of  $10^{-4} \text{ fm}^3$ . For the proton, the first error is the combined statistical and systematic uncertainty based on combining the results of several experiments, and the second represents an estimated theoretical error based on the model dependence in the extraction of the polarizabilities from the Compton scattering cross sections [5]. For the neutron, the first error is statistical and the second is systematic. While there remain nonnegligible experimental uncertainties, it appears likely that the neutron and proton magnetic polarizabilities are nearly identical while the electric polarizability of the neutron is slightly larger than that of the proton. The former result is not unexpected. However, the latter is somewhat of a surprise, at least in the context of a simple nonrelativistic valence quark model for the nucleon. In such a model, the polarizabilities are given by [6]

$$\bar{\alpha}_E = \alpha_E + \Delta\alpha_E, \quad \bar{\beta}_M = \beta_M + \Delta\beta_M, \tag{2}$$

where

$$\begin{aligned}\alpha_E &= 2\alpha \sum_{n \neq 0} \frac{|\langle n | \sum_i e_i (\mathbf{r}_i - \mathbf{R}_{CM})_z | 0 \rangle|^2}{E_n - E_0} \\ \beta_M &= 2\alpha \sum_{n \neq 0} \frac{|\langle n | \sum_i \frac{e_i}{2m_i} (\sigma_{iz} + L_{iz}) | 0 \rangle|^2}{E_n - E_0},\end{aligned}\tag{3}$$

and

$$\begin{aligned}\Delta\alpha_E &= \frac{Q\alpha}{3M} \langle \sum_i e_i (\mathbf{r}_i - \mathbf{R}_{CM})^2 \rangle \\ \Delta\beta_M &= -\frac{\alpha}{2M} \langle \left( \sum_i e_i (\mathbf{r}_i - \mathbf{R}_{CM}) \right)^2 \rangle - \frac{\alpha}{6} \langle \sum_i \frac{e_i^2 (\mathbf{r}_i - \mathbf{R}_{CM})^2}{m_i} \rangle,\end{aligned}\tag{4}$$

where  $Q$  is 1 for the proton and 0 for the neutron. In Eq. (3) the sum rule component, which is usually called the paramagnetic polarizability, receives its most important contribution from the Delta intermediate state, which is an isovector excitation and therefore contributes equally to the neutron and proton. One finds, including only this contribution [7]

$$\beta_M^{\circ,p} = \beta_M^{\circ,n} \approx 13. \quad (5)$$

The term  $\Delta\beta_M$ , which is usually called the diamagnetic polarizability, can be estimated within a simple nonrelativistic valence constituent quark model, with Hamiltonian

$$H = \frac{1}{2m}(\mathbf{p}_1^2 + \mathbf{p}_2^2 + \mathbf{p}_3^2) + \frac{m\omega_0^2}{2}(\mathbf{r}_{12}^2 + \mathbf{r}_{13}^2 + \mathbf{r}_{23}^2), \quad (6)$$

resulting in [8]

$$\Delta\beta_M = -\frac{\alpha}{m\omega_0} \begin{Bmatrix} \frac{2}{3} & p \\ \frac{5}{9} & n \end{Bmatrix} = -\alpha\langle r_p^2 \rangle \begin{Bmatrix} \frac{2}{3} & p \\ \frac{5}{9} & n \end{Bmatrix}$$

where  $\langle r_p^2 \rangle = \frac{2}{3} \frac{1}{m\omega_0}$  is the proton charge radius. (7)

Such an approach is clearly unrealistic. Indeed in such a picture the nucleon and Delta are degenerate and the neutron has zero charge radius. However, if a spin-spin interaction is included these problems can be ameliorated but Eq. (7) is only slightly affected[8]:

$$\Delta\beta_M = -\alpha(\langle r_p^2 \rangle - \langle r_n^2 \rangle) \times \begin{Bmatrix} \frac{2}{3} & p \\ \frac{5}{9} & n \end{Bmatrix} \approx \begin{Bmatrix} -10.2 & p \\ -8.5 & n \end{Bmatrix} \quad (8)$$

Thus we anticipate

$$\bar{\beta}_M^p - \bar{\beta}_M^n \approx 1.7. \quad (9)$$

Combining Eq. (5) and Eq. (8), we find

$$\bar{\beta}_M \approx \begin{Bmatrix} 2.8 & p \\ 4.5 & n \end{Bmatrix} \quad (10)$$

in reasonable accord with experiment. However, this agreement should not be overemphasized, as the Delta is only the most important of a large number

of possible intermediate states and the use of a simple valence quark model is also open to question.

What *is* a problem is the electric polarizability in this simple model, for which the recoil contribution is given by <sup>1</sup>

$$\Delta\alpha_E = \frac{\alpha}{3M} \begin{Bmatrix} \langle r_p^2 \rangle & p \\ 0 & n \end{Bmatrix} = \begin{Bmatrix} 3.6 & p \\ 0 & n \end{Bmatrix} \quad (11)$$

while the sum rule in Eq. (3) gives[15]

$$\alpha_E^p = \alpha_E^n = \frac{2\alpha}{3\bar{\omega}} \langle r_p^2 \rangle \approx 10.8, \quad (12)$$

where we have used a closure approximation and an average nucleon excitation energy of  $\bar{\omega} \approx 600$  MeV. The equality between the neutron and proton values is a result of charge symmetry, which requires that the valence quark excitations lead to identical excited states and p,n matrix elements. The precise value of the sum, which is difficult to calculate, then cancels out when we take the difference

$$\bar{\alpha}_E^n - \bar{\alpha}_E^p \approx \Delta\alpha_E^n - \Delta\alpha_E^p = -3.6. \quad (13)$$

This expectation, however, is in strong opposition to the experimental indication that

$$\bar{\alpha}_E^n - \bar{\alpha}_E^p \geq 0. \quad (14)$$

In fact, this difficulty is just another example of a well-known problem with valence quark models for the structure of the nucleon: namely, chiral symmetry is badly broken in such models because of the omission of mesonic degrees of freedom. By including only valence quark excitations in the sum over intermediate states, we are forced into the conclusion that the electric polarizability of the proton exceeds that of the neutron, in direct disagreement with experiment. In fact, it is well known that in the threshold region, the pion photoproduction on the nucleon is primarily nonresonant and that the cross section on the neutron is about 30% larger than that on the proton, a result which can easily be understood from a consideration of the effective

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<sup>1</sup>In a relativistic treatment,  $\Delta\alpha_E$  has an additional contribution  $\frac{\alpha(\lambda^2+Q)}{4M^3}$ , where  $\lambda$  is the anomalous magnetic moment [9]. Numerically, this extra term is 0.71 and 0.62 for the proton and neutron, respectively.

dipole moment of the  $\pi N$  system [11]. Similar considerations lead us to expect that  $\alpha_E$  for the neutron will exceed that for the proton. This qualitative idea is supported in part by a calculation in the context of the cloudy bag model [10], where it was shown that both the electric and the diamagnetic polarizabilities are dominated by the polarization of the pion cloud relative to the quark core and have very little contribution from the polarization of the core itself. This would lead one to expect the neutron electric polarizability to exceed that of the proton, although a definitive quantitative calculation in that model is not possible. A reasonable estimate *is* possible in chiral perturbation theory, where the only degrees of freedom that matter are pionic, and a recent one-loop calculation yields results in reasonable agreement with experiment [12]

$$\begin{aligned}(\alpha_E^n - \alpha_E^p)^{xpt} &= 3.1 \\ (\beta_M^n - \beta_M^p)^{xpt} &= 0.3.\end{aligned}\tag{15}$$

However, here too a rigorous evaluation is not available, as inclusion of important contributions such as the Delta are two-loop in character and are outside the present calculational framework [13].

We conclude then that a simple valence quark picture of the nucleon is in disagreement with experiment and that inclusion of meson cloud effects is *required* in order to understand the result that  $\bar{\alpha}_E^n > \bar{\alpha}_E^p$ . This finding is similar to that in the interpretation of the  $\langle N | \bar{s} \gamma_\mu \gamma_5 s | N \rangle$  matrix element, which also vanishes in a valence quark model but can be understood qualitatively by inclusion of a kaon cloud via  $N \rightarrow \Lambda K \rightarrow N$  [14]. However, a reliable calculation of the polarizability and of the strangeness content in this fashion is not possible.

Instead we follow a completely different approach, that of dispersion relations. On the one hand, this technique is capable of complete rigor in that the relations depend only on unitarity and certain analytic properties of the Compton scattering amplitudes. On the other hand, it is semi-phenomenological in that the evaluation of the dispersion integrals requires as input either experimental data or some reasonable theoretical *ansatz* when the required data are not available. In the next section we will present our results on the evaluation of the so-called backward dispersion relation for  $\bar{\alpha}_E - \bar{\beta}_M$ . Then we show how this dispersion relation can be used to calculate the contribution of the polarizability of the pion to that of the nucleon. Our conclusions are summarized in the concluding section.

## 2 Dispersion Sum Rules for the Polarizabilities

By combining dispersion relations with low energy theorems for the Compton scattering amplitudes, one can derive sum rules for the polarizabilities. A comprehensive review of this subject has been given by Petrun'kin [15]. The best known sum rule, the so-called Baldin-Lapidus sum rule, is based on the forward dispersion relation for the spin-independent part of the Compton scattering amplitude [16]:

$$\bar{\alpha}_E + \bar{\beta}_M = \frac{1}{2\pi^2} \int_0^\infty \frac{d\omega \sigma_{\text{tot}}(\omega)}{\omega^2} = \begin{cases} (14.2 \pm 0.5) & [\text{proton}] \\ (15.8 \pm 0.5) & [\text{neutron}], \end{cases} \quad (16)$$

In this expression,  $\sigma_{\text{tot}}$  is the total photoabsorption cross section, and the numerical values are based on the tabulations of those cross sections for the proton and neutron [17, 18]. The numbers given in Eq. (16) were actually used as a constraint in obtaining the experimental results in Eq. (1), so that those results do not test this sum rule. However, it *is* possible to reanalyze the recent data for the proton without imposing the sum rule constraint, in which case one obtains [5]

$$\bar{\alpha}_E^p + \bar{\beta}_M^p = 12.0 \pm 2.3 [\text{experiment}], \quad (17)$$

verifying the sum rule at the 1-standard deviation level.

It is also possible to write down sum rules for the difference of the electric and magnetic polarizabilities. The one we consider here is the so-called backward sum rule, which is based on a  $180^\circ$  dispersion relation and has the form[19]:

$$\bar{\alpha}_E - \bar{\beta}_M = (\text{s-channel contribution}) + (\text{t-channel contribution}). \quad (18)$$

The s-channel contribution is similar to Eq. (16), with a relativistic correction and with contributions from excitations with opposite parity entering with opposite sign—

$$\text{s-channel contribution} = \frac{1}{2\pi^2} \int_0^\infty \frac{d\omega}{\omega^2} \left(1 + \frac{2\omega}{m}\right)^{\frac{1}{2}} [\sigma_{\text{tot}}(\Delta P=\text{YES}) - \sigma_{\text{tot}}(\Delta P=\text{NO})], \quad (19)$$

where  $\sigma_{\text{tot}}(\Delta P=\text{YES})$  and  $\sigma_{\text{tot}}(\Delta P=\text{NO})$  represent those pieces of  $\sigma_{\text{tot}}$  arising from multipoles which change and do not change parity, respectively. The t-channel contribution can be written as

$$\begin{aligned} \text{t-channel contribution} = & \frac{1}{64\pi^2} \int_{4m_\pi^2}^{\infty} \frac{dt}{t^2} \sqrt{\frac{t-4m_\pi^2}{t}} \int d\Omega \left[ A^{(+)}(t, \cos \theta) \right. \\ & \left. + im \sqrt{\frac{t-4m_\pi^2}{4m^2-t}} \cos \theta B^{(+)}(t, \cos \theta) \right] F_0^*(t, \cos \theta), \quad (20) \end{aligned}$$

which corresponds to the approximation of including only the  $N\bar{N} \rightarrow \pi\pi \rightarrow \gamma\gamma$  intermediate state. The integration variable  $t$  is the square of the total center-of-mass energy of the  $\gamma\gamma$  system. Here  $A^{(+)}, B^{(+)}$  are the conventional CGLN isospin-even amplitudes for  $N\bar{N} \rightarrow \pi\pi$  [20],  $F_0(t, \cos \theta)$  is the  $I=0$  Gourdin-Martin  $\gamma\gamma \rightarrow \pi\pi$  amplitude [21], and  $m$  and  $m_\pi$  are the nucleon and pion masses, respectively. Because of the restriction to isoscalar amplitudes required by G-parity invariance, only even partial waves are permitted. Including only S- and D-waves, Eq. 20 simplifies to the form

$$\begin{aligned} \text{t-channel contribution} = & \frac{1}{16\pi^2} \int_{4m_\pi^2}^{\infty} \frac{dt}{t^2} \frac{16}{4m^2-t} \sqrt{\frac{t-4m_\pi^2}{t}} \left[ f_+^0(t) F_0^{0*}(t) \right. \\ & \left. - (m^2 - \frac{t}{4}) (\frac{t}{4} - m_\pi^2) f_+^2(t) F_0^{2*}(t) \right], \quad (21) \end{aligned}$$

where the partial wave helicity amplitudes  $f_+^J(t)$  for  $N\bar{N} \rightarrow \pi\pi$  are given by Frazer and Fulco [22] while the corresponding partial wave amplitudes  $F_0^J(t)$  for  $\gamma\gamma \rightarrow \pi\pi$  are defined in Ref. [21].

The backward sum rule has been previously evaluated by several authors [19, 23, 24]. However, several recent developments have renewed interest in this sum rule and have motivated us to perform a reanalysis with an eye towards a meaningful confrontation with the new experimental values. Such a confrontation is now possible because, as we will point out shortly, developments in understanding of the  $\gamma\gamma \rightarrow \pi\pi$  process has removed a major uncertainty in the calculation. Also the recent recognition of the importance of pions has rekindled interest in the relationship between the nucleon and pion polarizabilities [24], an issue that we will specifically address later in this paper. We now describe in detail our calculation.

In principle the s-channel integral is straightforward to calculate—provided one knows the multipole decomposition of  $\sigma_{\text{tot}}$ , one can separate



the  $\Delta P=\text{YES}$  from the  $\Delta P=\text{NO}$  contributions. Such a decomposition has been performed, however, only for the  $\pi N$  final state, which we therefore treat separately from the multi-pion final states. For this  $\pi N$  final state, which dominates  $\sigma_{\text{tot}}$  below 500 MeV, we use the multipoles of the VPI&SU group [25] and include  $\pi N$  partial waves through  $L=4$ . The integrands for the proton are shown in Fig. ??, and the corresponding integrands for the neutron are quite similar. Integrating up to 1800 MeV, we obtain the results

$$\begin{aligned}\bar{\alpha}_E^p - \bar{\beta}_M^p &= \begin{cases} +4.80 & [\text{s-channel single pion- } \Delta P=\text{YES}] \\ -10.78 & [\text{s-channel single pion- } \Delta P=\text{NO}] \end{cases} \\ \bar{\alpha}_E^n - \bar{\beta}_M^n &= \begin{cases} +6.04 & [\text{s-channel single pion- } \Delta P=\text{YES}] \\ -11.31 & [\text{s-channel single pion- } \Delta P=\text{NO}] \end{cases} \end{aligned} \quad (22)$$

For the multi-pion contribution, a precise calculation is not possible since an experimental multiple decomposition has not yet been performed. Nevertheless, one can establish rigorous bounds on that contribution in the following manner. At any given energy the entire multi-pion contribution to  $\sigma_{\text{tot}}$  can be determined by subtracting the *calculated* value of the single-pion contribution (using the VPI&SU multipoles) from the *full* experimental total photoabsorption cross section. Of course, this multi-pion piece is presumably associated with both  $\Delta P=\text{YES}$  and  $\Delta P=\text{NO}$  multipoles, and these two components contribute with opposite signs to the dispersion integral. We can obtain an upper or lower bound to the contribution of the multi-pion final states by assuming that the multi-pion photoabsorption is *completely*  $\Delta P=\text{YES}$  or  $\Delta P=\text{NO}$ , respectively. We have applied this procedure using two different compilations of the experimental total photoabsorption cross-section, one due to Damashek and Gilman[17] and the other due to Armstrong [26]. These give similar results for the dispersion integral. The integrand is shown in Fig. ?. In this way we find for the s-channel multi-pion contribution

$$\bar{\alpha}_E - \bar{\beta}_M = \pm 3.0 \quad [\text{s-channel multi-pion}], \quad (23)$$

where the positive sign applies if the multi-pion photoabsorption is purely  $\Delta P=\text{YES}$  (such as would obtain if the multi-pion part were principally  $\pi\Delta$  production in a relative S-state) and the negative sign applies if it is purely  $\Delta P=\text{NO}$  (such as would obtain if the multi-pion part were principally non-resonant  $\pi\pi N$  with everything in a relative S-state). The value quoted is for the proton since only in this case is there available a full tabulation of the

total photoabsorption cross section. However, it is reasonable to assume that the neutron contribution would be similar.

In the absence of additional experimental information on the multipole content of the multi-pion final states, the only way to improve on the above bounds is in the context of a model. One such model is that due to L'vov [18, 27], wherein  $\pi\pi N$  production is approximated by the pion pole contribution to the process  $\gamma N \rightarrow \pi\Delta \rightarrow \pi\pi N$ , and the amplitudes for all but the relative  $\pi\Delta$  S-wave are calculated in the Born approximation. The S-wave component is adjusted so that the total  $\pi\pi N$  cross section evaluated in this manner agrees with experiment [27]. Using the computer code supplied by L'vov, we have calculated these amplitudes and used them as input to the s-channel integral, the integrand of which is shown in Fig. ???. We find

$$\bar{\alpha}_E - \bar{\beta}_M = \begin{cases} +1.66 & [\text{s-channel multi-pion- } \Delta P=\text{YES}] \\ -1.10 & [\text{s-channel multi-pion- } \Delta P=\text{NO}], \end{cases} \quad (24)$$

with identical values for the neutron and proton, since such an approach yields a strictly isoscalar amplitude. We first note that the sum of the magnitudes of the two contributions (2.76) is slightly less than the value of 3.0 obtained above. Presumably this is due to the neglect of final states with three or more pions in the model. We further note that there is considerable cancellation between the  $\Delta P=\text{YES}$  and  $\Delta P=\text{NO}$  components in the model calculation, so that the net contribution of the multi-pion final states is quite small (0.56). We return to this point below when we compare with experiment.

For the t-channel integral, we require the amplitudes for both  $\pi\pi \rightarrow N\bar{N}$  and  $\gamma\gamma \rightarrow \pi\pi$ . The former can be reliably obtained by extrapolation from the cross-channel  $\pi N \rightarrow \pi N$  process as done by Bohannon and Signell [28], from which we take the amplitudes  $f_+^{0;2}(t)$  for use in Eq. (21). Previous calculations of the backward sum rule have utilized these same forms. The amplitudes for  $\gamma\gamma \rightarrow \pi\pi$  have traditionally been considered less reliable, especially for the S-wave part. However, in recent years there has been renewed interest in this reaction from both the experimental and theoretical side. In particular, new calculations lead to cross sections that are in excellent agreement with experimental data for both the  $\pi^+\pi^-$  and  $\pi^0\pi^0$  channels [29]. These calculations utilize dispersion relations with subtraction constants fixed by low energy theorems, taking into account the  $\pi\pi$  scattering phase shifts as

well as the effects of  $\pi$ ,  $\rho$ ,  $\omega$ , and  $A_1$  exchange [29]. At very low energy, the amplitude is dominated by the pion Born and polarizability terms,

$$F_0^0(t) = \frac{16\pi\alpha m_\pi^2}{\sqrt{t(t-4m_\pi^2)}} \ln \frac{t + \sqrt{t(t-4m_\pi^2)}}{t - \sqrt{t(t-4m_\pi^2)}} + 4\pi m_\pi t \bar{\alpha}_E^\pi + \dots \quad (25)$$

for S-wave and

$$F_0^2(t) = 40\pi\alpha \left[ -\frac{6m_\pi^2}{t-4m_\pi^2} + 2 \left( 1 + \frac{6m_\pi^2}{t-4m_\pi^2} \right) \frac{m_\pi^2}{\sqrt{t(t-4m_\pi^2)}} \ln \frac{t + \sqrt{t(t-4m_\pi^2)}}{t - \sqrt{t(t-4m_\pi^2)}} \right] \quad (26)$$

for D-wave. Here  $\bar{\alpha}_E^\pi$  is the electric polarizability of the charged pion, for which there is a precise prediction of  $(2.8 \pm 0.3)$  based on chiral symmetry, with parameters fixed from radiative pion decay [30]. The fact that neither the chiral prediction nor the  $\pi\pi \rightarrow \gamma\gamma$  results are expected to be accurate for energies  $E \geq 600$  MeV is not a significant problem as the factor  $t^{-2}$  in Eq. (21) guarantees rapid convergence of the dispersion integral (see Fig. ??). The contributions to the proton and neutron integrals are identical as only the isoscalar  $N\bar{N}$  channel is allowed by G-parity invariance.

In our numerical t-channel calculation, we include only S- and D-wave components. The integrand for the S-wave piece is given in Fig. ??, in which three different curves are shown, corresponding to three different representations of the  $\pi\pi \rightarrow \gamma\gamma$  amplitude: the Born approximation (the polarizability-independent term in Eq. (25)), Born plus pion polarizability (Eq. (25)), and the full dispersively calculated amplitude. The chiral prediction for  $\bar{\alpha}_E^\pi$  is used. We see that the full amplitude looks significantly different from the other two, mainly because of a zero in  $F_0^0(t)$  near 400 MeV which arises due to the Omnes function for I=0  $\pi\pi$  scattering [29]. The numerical results

$$\bar{\alpha}_E - \bar{\beta}_M = \begin{cases} +16.1 & [\text{t-channel S-wave, Born}] \\ +19.1 & [\text{t-channel S-wave, Born} + \bar{\alpha}_E^\pi] \\ +10.3 & [\text{t-channel S-wave, full}] \end{cases} \quad (27)$$

are quite sensitive to the location of this zero, which explains much of the uncertainty in the previous calculations of this contribution. Nevertheless, the excellent agreement between the full amplitude and the recent cross section data [29] gives us confidence in our result.

For the D-wave piece we use the Born approximation for  $\pi\pi \rightarrow \gamma\gamma$  (Eq. (26)). The integrand is shown in Fig. ??; the integral

$$\bar{\alpha}_E - \bar{\beta}_M = -1.7 \quad [\text{t-channel D-wave}], \quad (28)$$

is significantly smaller than its S-wave counterpart, thereby providing some justification for the neglect of higher partial waves. We note that the magnitude of our D-wave contribution is nearly a factor of four smaller than that given by previous authors [19, 24]. We do not understand the origin of this discrepancy.

Putting everything together we arrive then at our final results

$$\begin{aligned} \bar{\alpha}_E^p - \bar{\beta}_M^p &= \begin{cases} 5.6 & [\text{upper bound}] \\ 3.2 & [\pi\Delta \text{ model}] \\ -0.4 & [\text{lower bound}] \end{cases} \\ \bar{\alpha}_E^n - \bar{\beta}_M^n &= \begin{cases} 6.3 & [\text{upper bound}] \\ 3.9 & [\pi\Delta \text{ model}] \\ 0.3 & [\text{lower bound}] \end{cases} \end{aligned} \quad (29)$$

These numbers are to be compared with the experimental values:

$$\bar{\alpha}_E - \bar{\beta}_M = \begin{cases} 8.4 \pm 2.1 & [\text{experiment p}] \\ 9.4 \pm 5.0 & [\text{experiment n}] \end{cases} \quad (30)$$

Taking into account the errors<sup>2</sup> on the experimental results, there is good overall consistency with the backward sum rule, provided the actual contribution of the s-channel multi-pion contribution is somewhere between the upper bound and the  $\pi\Delta$  model prediction. However, additional work would be very helpful in extending these findings. In particular a multipole analysis of the  $\gamma N \rightarrow \pi\pi N$  process, such as is presently planned at Argonne [31], would help to clarify the full s-channel dispersive analysis.

We now return to the point that originally motivated this work, the size of  $\bar{\alpha}_E^n$  relative to  $\bar{\alpha}_E^p$ . Combining the Baldin and backward sum rules, we obtain

$$\bar{\alpha}_E = \frac{1}{2} [(\bar{\alpha}_E + \bar{\beta}_M) + (\bar{\alpha}_E - \bar{\beta}_M)], \quad (31)$$

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<sup>2</sup>To obtain the error on  $\bar{\alpha}_E - \bar{\beta}_M$ , we first combine in quadrature the errors on  $\bar{\alpha}_E$  in Eq. (1), then double the result, since the errors on  $\bar{\alpha}_E$  and  $\bar{\beta}_M$  are anticorrelated [5].

and then take the neutron-proton difference of these quantities. The dispersion prediction should be particularly accurate for this difference because the principal uncertainties in our calculation are in the s-channel multi-pion contribution, which is approximately isoscalar, and the t-channel contribution, which is rigorously isoscalar. Therefore, those uncertainties are largely removed when we take the neutron-proton difference. We find the following:

$$\bar{\alpha}_E^n - \bar{\alpha}_E^p = \begin{cases} 1.2 & [\text{dispersion relations}] \\ 1.3 \pm 1.9 & [\text{experiment}] \\ -3.6 & [\text{valence quark model}] \end{cases} \quad (32)$$

We see that the dispersion theory does remarkably well in quantitatively accounting for the relative sizes of the electric polarizability for the neutron and proton. It appears that both the chiral perturbative, Eq. (15), and dispersive calculations are quite consistent with the experimental findings, while the simple constituent quark model, Eq. (13), is strongly at variance. We conclude that taking the pion cloud components of the nucleon into account is essential in order to understand the recent polarizability results.

### 3 Connecting Pion and Nucleon Polarizabilities

Since the electric and the diamagnetic polarizabilities are dominated by the pion cloud, it is reasonable to ask whether the intrinsic polarizability of the pion itself contributes to that of the nucleon. Intuitively such a connection is expected since the presence of an external electromagnetic field can not only polarize the pion cloud relative to the quark core but can also, to the extent that the pion is polarizable, polarize the pions themselves. The backward dispersion relation enables us to derive a model-independent relation between the nucleon and pion polarizabilities, which can be compared to the predictions of various models. In this section we address this issue.

Cohen and Broniowski have derived a quantitative relation between the nucleon and pion polarizabilities in the context of a hedgehog model of the nucleon [32]. They focus on the  $L_9, L_{10}$  component—the piece responsible for giving the pion electromagnetic structure—of the effective action describing the interaction of Goldstone bosons, as written down by Gasser and

Leutwyler[33]

$$\begin{aligned}\mathcal{L}_{\text{eff}} = \dots - iL_9 \text{Tr} \left[ F_{\mu\nu}^L D^\mu U D^\nu U^\dagger + F_{\mu\nu}^R D^\mu U^\dagger D^\nu U \right] \\ + L_{10} \text{Tr} \left[ F_{\mu\nu}^L U F^{R,\mu\nu} U^\dagger \right],\end{aligned}\quad (33)$$

where  $F_{\mu\nu}^{L,R}$  are the left, right chiral field strength tensors, which in the electromagnetic case take the form  $F_{\mu\nu}^{L,R} = \frac{e}{2}\tau_3 F_{\mu\nu}$ . In the linear sigma model,  $U$  describes the chiral field,  $U = \frac{1}{F_\pi}(\sigma + i\boldsymbol{\tau} \cdot \boldsymbol{\pi})$ . At tree level the charged pion polarizability can be completely described in terms of  $L_9, L_{10}$ :

$$\begin{aligned}L_9 &= \frac{F_\pi^2 \langle r_E^2 \rangle}{12} \\ L_{10} &= \frac{m_\pi F_\pi^2 4\pi \bar{\alpha}_E^\pi}{4e^2} - L_9 \equiv \frac{m_\pi F_\pi^2 4\pi \alpha_E^\pi}{4e^2} \\ &\text{with } \bar{\alpha}_E^\pi = -\bar{\beta}_M^\pi\end{aligned}\quad (34)$$

In a mean field approach, treating the meson operators as classical fields and taking  $\mathbf{E}, \mathbf{B}$  to be constants, one finds

$$\int d^3x \mathcal{L} \sim \int d^3x \frac{4e^2}{F_\pi^2} (L_9 + L_{10}) (\mathbf{E}^2 - \mathbf{B}^2) (\mathbf{c} \times \boldsymbol{\pi}_h)^2 \quad (35)$$

where  $\pi_h$  is the hedgehog pion field and  $\mathbf{c}$  is defined in ref. 32. The spatial integral can be related to the fraction of the total moment of inertia carried by the pion degree of freedom and yields the estimate[32]

$$\delta \bar{\alpha}_E^N = -\delta \bar{\beta}_M^N \approx 0.5 \bar{\alpha}_E^\pi, \quad (36)$$

where  $\delta \bar{\alpha}_E^N$  refers to that part of the nucleon polarizability that is due to the intrinsic polarizability of the pion.

It is possible to understand this result in an alternative fashion, using Feynman diagrams. Thus the effective charged pion electromagnetic interaction due to its polarizability can be written in the local form[34]

$$\mathcal{L}_{\text{eff}}^\pi = \frac{1}{4} F_{\mu\nu} F^{\mu\nu} 4\pi \bar{\alpha}_E^\pi 2m_\pi \pi^+ \pi^-. \quad (37)$$

Insertion into the diagram shown in Fig. ?? then yields

$$\begin{aligned}\mathcal{L}_{\text{eff}}^N &= \frac{1}{4} F_{\mu\nu} F^{\mu\nu} 4\pi \bar{\alpha}_E^\pi 2m_\pi (\sqrt{2}g)^2 \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - m_\pi^2)^2} \bar{\psi} \gamma_5 \frac{1}{\gamma^\mu (p-k)_\mu - m_N} \gamma_5 \psi \\ &= F_{\mu\nu} F^{\mu\nu} \bar{\psi} \psi \times 4\pi \bar{\alpha}_E^\pi \left(\frac{g}{4\pi}\right)^2 r_\pi I(r_\pi^2)\end{aligned}\quad (38)$$

where  $r_\pi = m_\pi/m_N$  and

$$I(x) = \int_0^1 dy \frac{y(1-y)}{y^2 + x(1-y)}. \quad (39)$$

We identify then the contribution to the nucleon polarizability due to the analogous pion polarizability as

$$\delta\bar{\alpha}_E^N = -\delta\bar{\beta}_M^N = 4\left(\frac{g}{4\pi}\right)^2 r_\pi I(r_\pi^2) \bar{\alpha}_E^\pi = 0.8\bar{\alpha}_E^\pi, \quad (40)$$

which is somewhat larger than the hedgehog number. However, it must be emphasized that this is a simple one loop calculation and must therefore be considered to be only a crude estimate.

Finally, we derive a basically model-independent result based on the backward dispersion relation. The connection comes via the t-channel integral, Eq. (21), and the low-energy form of the S-wave part of the  $\gamma\gamma \rightarrow \pi\pi$  amplitude, Eq. (25), from which one easily derives

$$\delta(\bar{\alpha}_E^N - \bar{\beta}_M^N) = \bar{\alpha}_E^\pi \frac{4m_\pi}{\pi} \int_{4m_\pi^2}^\infty \frac{dt \sqrt{t - 4m_\pi^2}}{t^{\frac{3}{2}}(4m_N^2 - t)} |f_+^0(t)|, \quad (41)$$

Numerical evaluation of this integral then gives<sup>3</sup>

$$\delta(\bar{\alpha}_E^N - \bar{\beta}_M^N) = 1.01\bar{\alpha}_E^\pi \quad (42)$$

Since from our previous discussion the pion contribution to the nucleon electric/magnetic polarizabilities is equal and opposite, we can rewrite Eq. (42) as

$$\delta\bar{\alpha}_E^N = -\delta\bar{\beta}_M^N = 0.5\bar{\alpha}_E^\pi \quad (43)$$

which is a rigorous result and in satisfactory agreement with the estimates provided above via hedgehog and Feynman diagram arguments.

The size of this contribution to the nucleon polarizability depends upon the size of the charged pion polarizability, whose value is still experimentally uncertain. Although chiral symmetry makes a rather firm theoretical prediction [30]

$$\bar{\alpha}_E^\pi = -\bar{\beta}_M^\pi = \frac{4\alpha(L_9 + L_{10})}{m_\pi F_\pi^2} = 2.8 \text{ [chiral prediction]}, \quad (44)$$

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<sup>3</sup>This procedure has been looked at previously by V.M. Budnev and V.A. Karnakov [24]. However, there appear to exist a number of serious dimensional errors in their paper (*cf.* Eqs. 4 and 9) so that the numerical values given therein must be questioned.

the experimental situation is yet unclear with three different results being provided by three very different techniques:

$$\bar{\alpha}_E^\pi = \begin{cases} 2.2 \pm 1.1 & [\gamma\gamma \rightarrow \pi\pi] & [35] \\ 6.8 \pm 1.4 & [\text{radiative pion scattering}] & [36] \\ 20 \pm 12 & [\text{radiative pion photoproduction}] & [37] \end{cases} \quad (45)$$

Comparison with the recently measured nucleon values Eq. (1) indicates that the pion contribution to the nucleon polarizability is relatively modest if the chiral prediction or the  $\gamma\gamma \rightarrow \pi\pi$  result is correct, but is rather significant if the radiative pion scattering value were to be correct. Note that since the t-channel dispersive piece is isoscalar, its contribution to neutron and proton values is identical. Clearly it is important to remeasure the pion polarizability in order to resolve the origin of these discrepant values, and such efforts are planned at Brookhaven, Fermilab, and DaΦne.

## 4 Conclusions

Recent experimental measurements of the nucleon electromagnetic polarizabilities are shown to be inconsistent with expectations based on a simple constituent quark model picture of the nucleon—mesonic contributions must be included in order to understand these findings. Dispersion relations offer a rigorous approach to this problem, but depend sensitively upon the correctness of the s- and t-channel integrands. Considerable recent progress has been made in this regard. In the case of the t-channel, successful dispersive/ chiral perturbative analyses of the  $\gamma\gamma \rightarrow \pi\pi$  reaction have enabled a reasonably reliable estimate of this contribution, while in the case of the s-channel a multipole analysis of the  $N\pi$  intermediate state enables a believable calculation of this piece. Further progress awaits a similar multipole decomposition of the (smaller) multi-pion component as well as an improvement on the precision of the neutron polarizability measurements. However, overall agreement between the experimental numbers and the dispersive predictions must be judged to be quite satisfactory. Finally, we have used the t-channel part of the dispersion relation to do a precise calculation of the contribution of the pion polarizability to that of the nucleon.

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## Figure Captions

Figure 1: Integrand of the s-channel contribution for the proton. The solid/dashed curves are the integrands for the single-pion parity changing/non-changing multipoles, respectively. The dotted curve is the integrand obtained by subtracting the single pion from the total photoproduction cross section, and the integral of that curve gives a rigorous bound on the multi-pion contribution.

Figure 2: The integrand of the s-channel multipion contributions for the proton, as given by the  $\pi\Delta$  model of L'vov. The solid/dashed curves are the integrands for the two-pion parity changing/non-changing multipoles, respectively.

Figure 3: The integrand for the t-channel S-wave contribution. The solid, dotted, and dashed curves correspond to the Born, Born+pion polarizability and full amplitudes, respectively, for the  $\gamma\gamma \rightarrow \pi\pi$  process.

Figure 4: The integrand for the t-channel D-wave contribution.

Figure 5: Pion-exchange diagram contributing to the nucleon polarizability.

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